MATHEMATICAL LOGIC

Place and time: In M105 on Friday, Jan 5, at 16:00–17:30
Organizer: Juha Kontinen (University of Helsinki)
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Categoricity in Universal Classes

Kaisa Kangas *(University of Helsinki)*, `kaisa.kangas@helsinki.fi`

Abstract. Let $(\mathcal{K}, \subseteq)$ be a universal class with $LS(\mathcal{K}) = \lambda$ categorical in regular $\kappa > \lambda^+$ with arbitrarily large models, and let $\mathcal{K}^*$ be the class of all $\mathcal{A} \in \mathcal{K}_{>\lambda}$ for which there is $\mathcal{B} \in \mathcal{K}_{\geq\kappa}$ such that $\mathcal{A} \subseteq \mathcal{B}$. We prove that $\mathcal{K}^*$ is categorical in every $\xi > \lambda^+$, and the models of $\mathcal{K}^*_{>\lambda^+}$ are essentially vector spaces (or trivial i.e. disintegrated).

Joint work with T. Hyttinen.

On Approximations and Eigenvectors — a look at Quantum Physics via Metric Ultraproducts

Åsa Hirvonen *(University of Helsinki)*, `asa.hirvonen@helsinki.fi`

Abstract. Quantum mechanics is usually modelled via self adjoint operators in the complex Hilbert space $L_2(\mathbb{R})$. There is a tradition of approximating calculations in this model using finite dimensional complex Hilbert spaces. I present a model theoretic way of looking at such approximations, based on ultraproducts of metric structures, which makes precise the notion of approximation and its effect on calculations.

The ultraproduct allows one to define and calculate the Feynman propagator as the inner product $\langle x_0|K^t|x_1\rangle$, where $|x_i\rangle$ are eigenvectors of the position operator and K^t is the time evolution operator. The calculations use Gauss sums which, however, causes a discretising effect, giving the wrong value at the limit. This can be remedied by instead of the propagator looking at the kernel of the time evolution operator. Mathematically the propagator and the kernel are different things, but they are used the same way in calculating the movement of a particle and thus should have the same value. Calculating the limit of the kernels allows one to overcome the discretising effect and still use the benefits of finite Gauss sums.

[1] Hirvonen Å. and Hyttinen T., *On Eigenvectors and the Feynman Propagator*, (under revision).

Joint work with T. Hyttinen.

Model-Checking Games for Temporal Logics

Raine Rönnholm *(University of Tampere)*, `raine.ronnholm@uta.fi`

Abstract. A model-checking game (aka a semantic game) is played between two players: the verifier (*Eloise*) and the falsifier (*Abelard*). These games can be seen as formal arguments where Eloise claims that a given formula φ is true in a given model $\mathcal{M}$ and Abelard tries to prove her wrong. Game-theoretic semantics equates the truth of φ in $\mathcal{M}$ with the existence of a winning strategy for Eloise in the corresponding model-checking game.

We present natural model-checking games for temporal logics LTL, CTL, ATL and some of their extensions. We also define so-called "bounded" variants of these games to guarantee finite plays – even in infinite models. We study the corresponding game-theoretic semantics for these logics and make comparison with their standard (compositional) semantics. We also show how model-checking games relate to alternating Turing machines and how a game-theoretic approach can thus be applied for studying the complexity of model-checking.

Joint work with Valentin Goranko and Antti Kuusisto.