

# OPTIMIZATION WITH APPLICATIONS

*Place and time:* In M106 on Thursday, Jan 4, at 16:00–17:30  
*Organizers:* Napsu Karmitsa (University of Turku)  
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## Double Bundle Method for Nonsmooth DC Optimization

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**Abstract.** A class of functions presented as a difference of two convex (DC) functions constitutes an important subclass of nonconvex functions, since these functions preserve some important properties of convex functions. In addition, many practical problems can be expressed in a DC form such as location planning, engineering design and cluster analysis. However, a stopping condition used in most nonsmooth DC algorithms guarantees only criticality for the solution point and this condition is weaker than Clarke stationarity typically used in general nonconvex nonsmooth optimization.

We will introduce a new double bundle method for unconstrained nonsmooth DC minimization utilizing explicitly the DC decomposition of the objective. The main novelty of the method is a new procedure designed to ensure Clarke stationarity for candidate solutions by using only information about the DC components. In addition, if a candidate solution is not Clarke stationary, then the procedure yields a descent direction. In practice, this means that we are able to avoid some drawbacks encountered when criticality is used as a stopping condition. Some encouraging numerical results will also be presented.

*Joint work with A.M. Bagirov, N. Karmitsa, M.M. Mäkelä and S. Taheri.*

## Multi-marginal Optimal Transport

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**Abstract.** Over the past ten years, multi-marginal optimal transport, a generalization of the well known optimal transport problem of Monge and Kantorovich, has begun to attract considerable attention, due in part to a wide variety of emerging applications.

In this talk, we give a short overview on theoretical and numerical aspects of Optimal Transport Theory for finitely many marginals, including applications. We hope to point out many theoretical and numerical challenges, including the case of Coulomb-type costs which are important for Density Functional Theory.

## Data-driven decision support with multiobjective optimization and a case in inventory management

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**Abstract.** Thanks to digitalization, we have access to various types of data and must decide how to make the most of the data. We can use descriptive or predictive analytics but to make recommendations and informed decisions based

on the data, we need prescriptive or decision analytics. If the problems contain multiple conflicting objectives, multiobjective optimization are to be applied.

Lot sizing is an example of a data-driven optimization problem. It is important in production planning and inventory management, where a decision maker needs support, in particular, when the demand is stochastic. We consider the lot sizing problem of a Finnish production company and formulate four conflicting objectives. We solve it with two interactive multiobjective optimization methods. In interactive methods, a decision maker directs the search for the best balance between the conflicting objectives by providing preference information. In this way, (s)he can learn about what kind of solutions are available for the problem and also learn about the feasibility of one's preferences.

In the case considered, the decision maker found it useful to switch the method during the solution process. The results of this data-driven interactive multiobjective optimization approach are encouraging and demonstrate the practical value of decision analytics.

*Joint work with J. Sipilä, V. Ojalehto and R. Heikkinen.*