

# MATHEMATICAL BIOLOGY III

*Place and time:* In M100 on Friday, Jan 5, at 10:30–12:00  
*Organizers:* Mats Gyllenberg (University of Helsinki)  
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## The Kimura Equation

FABIO CHALUB (*Universidade Nova de Lisboa*), [facc@fct.unl.pt](mailto:facc@fct.unl.pt)

**Abstract.** The Kimura Equation was introduced in the 60's by the Japanese geneticist Motoo Kimura and is considered one of the most important models in population genetics. It is a degenerated partial differential equation of drift diffusion type modelling the evolution of the probability distribution among different genotypes in a population.

In this talk we will derive this equation from basic stochastic models, showing not only that it approximates in all time scales important models as the Moran and the Wright-Fisher models but that it also encloses the well know replicator equation (a first order ordinary differential equation used extensively in evolutionary game theory). We will also show that the correct formulation of the Kimura equation includes two linearly independent conservation laws to be satisfied at all times.

In the final part, we will discuss generalizations and new formulations of the same problem.

## Attractive-repulsive models in collective behavior and applications

JOSÉ ANTONIO CARRILLO (*Imperial College London*), [carrillo@imperial.ac.uk](mailto:carrillo@imperial.ac.uk)

**Abstract.** We will discuss properties of solutions to aggregation-diffusion models appearing in many biological models such as cell adhesion, organogenesis and pattern formation. We will concentrate on typical behaviours encountered in systems of these equations assuming different interactions between species under a global volume constraint.

## Density-dependent diffusion and pattern formation

STEFAN GERITZ (*University of Helsinki*), [stefan.geritz@helsinki.fi](mailto:stefan.geritz@helsinki.fi)

**Abstract.** Diffusion in an activator-inhibitor system on a domain with reflecting boundaries can lead to the onset of spatial pattern formation by destabilizing the spatially uniform equilibrium, known as Turing instability. Several other mechanisms for pattern formation have been described in the literature as well. In this presentation we introduce a mechanism for pattern formation that is purely based

on density-dependent diffusion in a single species system. To this end we first look at particle movement along a linear array of well-mixed compartments separated by semipermeable barriers as a microscopic model of the diffusion process. On the macroscopic scale, we find that destabilization of the uniform equilibrium (and hence pattern formation) is possible depending on whether, in the microscopic model, local population density affects the barrier hitting rate per particle or the barrier crossing probability per hit, as well as on the strength and sign of the effect.

*Joint work with F. Wei.*