

# PARTIAL DIFFERENTIAL EQUATIONS

*Place and time:* In M103 on Friday, Jan 5, at 16:00–17:30  
*Organizer:* Mikko Parviainen (University of Jyväskylä)  
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Regularity properties for elliptic equations in non-divergence form  
AMAL ATTOUCHI (*University of Jyväskylä*), amal.a.attouchi@jyu.fi

**Abstract.** The regularity theory of fully nonlinear uniformly elliptic equations is well developed. However, it is not straightforward to extend this theory to the case of possibly degenerate or singular quasilinear equations. In this talk we discuss regularity properties of viscosity solutions of some particular elliptic equations in non-divergence form modeled on the  $p$ -Laplacian. More precisely, we focus on viscosity solutions of

$$-|Du|^\gamma \Delta_p^N u := -|Du|^{\gamma-(p-2)} \Delta_p u = f$$

where  $\gamma > -1$ ,  $1 < p < \infty$  and  $f$  is continuous and bounded. We review some recent results and provide local Hölder estimates for the gradient in the full range  $p > 1$ ,  $\gamma > -1$ , and local  $W^{2,2}$  estimates for a restricted range where  $p$  is close to 2 and  $\gamma$  is close to 0.

*Joint work with E. Ruosteenoja.*

Hölder continuity in time for linear parabolic systems  
OLLI SAARI (*University of Bonn*), saari@math.uni-bonn.de

**Abstract.** Weak solutions to elliptic partial differential equations with measurable coefficients are locally Hölder continuous by the de Giorgi–Nash–Moser theory. In the case of systems, no regularity is to be expected (at least in dimensions higher than three), and the same goes for the pure space-variable regularity of solutions to parabolic systems. However, a general theorem due to Lions asserts that a solution to the parabolic problem is always a continuous mapping from a time interval into functions locally integrable in space. In this talk, I discuss how to improve the continuity in Lions’ theorem into Hölder continuity.

*Joint work with P. Auscher, S. Bortz and M. Egert.*

$C^{1,\alpha}$ -regularity of  $p$ -laplace equation in the Heisenberg Group  
SHIRSHO MUKHERJEE (*University of Jyväskylä*), shirsho.s.mukherjee@jyu.fi

**Abstract.** Analysis of sub-elliptic equations goes back to the seminal work of Hörmander in the 60’s. Since then, regularity of quasi-linear sub-elliptic equations, has been a subject of increasing interest and active research and has involved works of many authors.

In this talk, a proof of  $C^{1,\alpha}$ -regularity for the  $p$ -laplace equation in the Heisenberg Group for the full range  $1 < p < \infty$ , shall be discussed. This problem has remained open for last few decades.

*Joint work with X. Zhong.*