MATHEMATICAL CONTROL THEORY AND APPLICATIONS

Place and time:	In M106 on Friday, Jan 5, at 16:00–17:30
Organizers:	Lassi Paunonen (Tampere University of Technology)
	Mikael Kurula (Åbo Akademi University)
Contact email:	`lassi.paunonen@tut.fi`

Webster's horn model on Bernoulli flow

Jarmo Malinen *(Aalto University)*, `jarmo.malinen@aalto.fi`

Abstract. Disregarding all viscosity and friction effects, the steady flow of an incompressible fluid in a variable, circular cross-section tube can be understood in terms of the classical Bernoulli principle. If the fluid, however, has some compressibility, it also supports acoustic vibrations. Considering only the longitudinal acoustics in stationary fluid constrained into such a tube, an efficient model is provided by Webster's horn equation, governing the averages of the acoustic field on the cross-sections of the tube.

The acoustics of a moving fluid in a tube deals with combination of the two kinds of effects that couple in a complicated way. For example, the acoustic field moves together with the fluid, and the speed of transport is accelerated at the constrictions of the tube.

We propose a simplified model in terms of a Partial Differential Equation, satisfied by a modified acoustic velocity potential, for the longitudinal acoustics in a moving fluid column. The equation is derived from the fundamental equations of fluid mechanics, and it reduces to the classical lossless Webster's equation when the underlying fluid velocity vanishes.

Model Predictive Control for Regular Linear Systems

Jukka-Pekka Humaloja *(Tampere University of Technology)*,
`jukka-pekka.humaloja@tut.fi`

Abstract. This talk considers model predictive control (MPC) for regular linear systems in the discrete-time setting. A continuous-time regular linear system is mapped to a discrete-time system using the Cayley-Tustin transform, and the MPC problem is solved for the discretized system. The considered MPC problem is of the form

$$
\begin{aligned}
&\min_u \sum_{i=1}^{\infty} \langle y_i, Q y_i \rangle + \langle u_i, R u_i \rangle \qquad (Q, R > 0 \text{ given}) \\
&\text{s.t.} \quad x_i = A_d x_{i-1} + B_d u_i \\
&\qquad\quad y_i = C_d x_{i-1} + D_d u_i \\
&\qquad u_{\min} \le u_i \le u_{\max} \qquad \forall i \in \mathbb{N} \\
&\qquad y_{\min} \le y_i \le y_{\max} \qquad \forall i \in \mathbb{N}
\end{aligned}
$$

which will be cast from the infinite-horizon formulation to a finite-horizon one by

using a state penalty term. The finite-horizon formulation results in a quadratic optimization problem in a convex domain, for which the global minimum can be easily found. The MPC methodology is demonstrated for the one-dimensional wave equation with boundary controls and observations.

Joint work with S. Dubljevic.

Efficient solution of symmetric eigenvalue problems related to coupled systems

ANTTI HANNUKAINEN *(Aalto University)*, antti.hannukainen@aalto.fi

Abstract. In this talk, I discuss efficient solution of symmetric eigenvalue problems

$$A\vec{x} = \lambda M\vec{x}, \tag{1}$$

related to systems consisting of two coupled subsystems. The motivation for this work arises from modal computations of a vocal tract constrained into an MRI machine. In this case, the system consists of the vocal tract air volume (i.e., *the interior system*) that changes during speech, and the air volume of the MRI machine (i.e., *the exterior system*) that stays unchanged. For understanding speech production, it is desirable to compute the resonance structure of the acoustic system for a large number of vocal tract shapes. In order to speed up these computations, there is a strong incentive to precompute the effect of the unchanged exterior system and use it efficiently.

The eigenvalue problem (1) is decomposed as

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \end{bmatrix} = \lambda \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \end{bmatrix} \tag{2}$$

where $\vec{x}_1$ and $\vec{x}_2$ are related to interior and exterior systems, respectively. I present a novel method for efficiently solving Eq. (2) for the smallest eigenvalues $\lambda \in [0, L]$, $L > 0$, when A_{22}, M_{22}, $range\, A_{21}$, and $range\, M_{21}$ remain unchanged for large number of different A_{11}. The number of eigenvalues in $[0, L]$ is typically much smaller than the dimension of the full problem. The proposed method allows dimension reduction and optimization of the matrices related to the exterior system.

Joint work with J. Malinen and A. Ojalammi.