# MATHEMATICAL BIOLOGY IV

*Place and time:* In M100 on Friday, Jan 5, at 16:00–17:30
*Organizers:* Mats Gyllenberg (University of Helsinki)
Eva Kisdi (University of Helsinki)
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## An extension of the classification of evolutionarily singular strategies in Adaptive dynamics

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**Abstract.** The basic framework of Adaptive dynamics assumes an invasion fitness that is differentiable twice as a function of both the resident and the invader trait. Motivated by nested models of infectious disease dynamics we consider an extended framework in which the selection gradient exists (so the definition of evolutionary singularities extends *verbatim*), but where invasion fitness may lack the smoothness necessary for the classification à la Geritz et al. [*Evol. Ecol.*, 12, pp. 3557 (1998)]. We present the classification of evolutionarily singular strategies with respect to convergence stability and invadability and determine the condition for existence of nearby dimorphisms. The extended setting of Adaptive dynamics allows for a new type of evolutionary singularity: a so called one-sided ESS that is invadable by mutant strategies on one side of the singularity but uninvadable by mutants on the other side. We discuss possible evolutionary scenarios nearby one-sided ESSs and conclude by applying the extended framework to nested models of infectious disease dynamics.

*Joint work with O. Diekmann.*

## The effect of spatial heterogeneity on evolution in spatial models

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**Abstract.** The Wright's island model consists of a large number of ecologically identical patches, in which a fixed number of adults produce offspring and die. Part of the offspring disperse. Those individuals surviving dispersal arrive randomly in any other patch. After dispersal, the $n$ individuals to become adults are randomly chosen among the offspring present in each patch.

We investigate an extension including spatial heterogeneity, so that patches can be of different quality. By investigating metapopulation fitness, we present analytical expressions for the selection gradient and conditions for convergence stability and evolutionary stability.

In the homogeneous model, evolutionary branching of dispersal is not possible [1]. We show that spatial heterogeneity selects against dispersal, but can promote evolutionary branching.

For a fecundity-affecting trait, Taylor's cancellation result holds in the homogeneous model: Not only singular strategies but also their convergence stability is identical to that in the corresponding well-mixed model. Homogeneous spatial structure also often inhibits evolutionary branching: Evolutionary branching never occurs when the dispersal rate is close to zero, and for a wide class of fecundity functions (including those determined by any pairwise game), evolutionary branching is impossible for any dispersal rate if branching does not occur in the corresponding well-mixed model [2].

In contrast, in a spatially heterogeneous model, evolutionary branching can happen for low dispersal rates, even when it does not happen when everybody disperses.

[1] É. Ajar. Analysis of disruptive selection in subdivided populations. *BMC Evolutionary Biology*, 3:22:1–12, 2003.

[2] Kalle Parvinen, Hisashi Ohtsuki, and Joe Wakano. The effect of fecundity derivatives on the condition of evolutionary branching in spatial models. *J. Theor. Biol*, 416:129–143, 2017.

*Joint work with Hisashi Ohtsuki and Joe Yuichiro Wakano.*

## Evolutionary suicide of prey: Matsuda and Abrams' model revisited

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**Abstract.** Evolution by natural selection is widely believed to perfect the species and to provide them the highest possible chance to survive. This naive view is now known to be incorrect; in particular, adaptive evolution can drive a population to its own extinction through a process called evolutionary suicide. The earliest model exhibiting evolutionary suicide is by Matsuda and Abrams (1994, Theor. Pop. Biol. 45: 76-91). They considered a species of prey evolving its foraging effort in the presence of predators, and showed that the prey, hiding from the predator, can decrease foraging until it crosses a point of a fold bifurcation, whereupon its population crashes to extinction. Here we revisit and extend this model in three directions. First, we generalize the trade-off function the model assumes between the prey's foraging effort and its fecundity. By a constructive proof, we show that it is always possible to find trade-off functions such that evolutionary suicide occurs via evolving lower foraging efforts, but never via higher foraging efforts, even though foraging may evolve both ways and a fold bifurcation can occur in both directions. Second, we extend the model to non-equilibrium systems and demonstrate evolutionary suicide at a fold bifurcation of limit cycles. Third, we embed the model into a 3-species community of the focal prey, its predator, and an alternative prey of the predator. In this 3-species model, we find rich dynamics including evolutionary suicide via a subcritical Hopf bifurcation.

*Joint work with C. Vitale.*