

INVERSE PROBLEMS II

Place and time: In M103 on Thursday, Jan 4, at 16:00–17:30
Organizers: Joonas Ilmavirta (University of Jyväskylä)
Jere Lehtonen (University of Jyväskylä)
Contact email: joonas.ilmavirta@jyu.fi

Geometric Whitney problem: Reconstruction of a manifold from a point cloud

MATTI LASSAS (*University of Helsinki*), matti.lassas@helsinki.fi

Abstract. We study the geometric Whitney problem on how a Riemannian manifold (M, g) can be constructed to approximate a metric space (X, d_X) . This problem is closely related to manifold interpolation (or manifold learning) where a smooth n -dimensional surface $S \subset \mathbb{R}^m$, $m > n$ needs to be constructed to approximate a point cloud in \mathbb{R}^m . These questions are encountered in differential geometry, machine learning, and in many inverse problems encountered in applications. The determination of a Riemannian manifold includes the construction of its topology, differentiable structure, and metric.

We give constructive solutions to the above problems. Moreover, we characterize the metric spaces that can be approximated, by Riemannian manifolds with bounded geometry: We give sufficient conditions to ensure that a metric space can be approximated, in the Gromov-Hausdorff or quasi-isometric sense, by a Riemannian manifold of a fixed dimension and with bounded diameter, sectional curvature, and injectivity radius. Also, we show that similar conditions, with modified values of parameters, are necessary.

Moreover, we characterise the subsets of Euclidean spaces that can be approximated in the Hausdorff metric by submanifolds of a fixed dimension and with bounded principal curvatures and normal injectivity radius.

The above interpolation problems are also studied for unbounded metric sets and manifolds. The results for Riemannian manifolds are based on a generalisation of the Whitney embedding construction where approximative coordinate charts are embedded in \mathbb{R}^m and interpolated to a smooth surface. We also give algorithms that solve the problems for finite data.

- [1] C. Fefferman, S. Ivanov, Y. Kurylev, M. Lassas, H. Narayanan: Reconstruction and interpolation of manifolds I: The geometric Whitney problem.
ArXiv:1508.00674

Joint work with C. Fefferman, S. Ivanov, Y. Kurylev, and H. Narayanan.

Recovering electric potential from backscattering measurements

MARKUS HARJU (*University of Oulu*), markus.harju@oulu.fi

Abstract. In this talk we consider the magnetic Schrödinger operator $H = -(\nabla + iW)^2 + V$ with real-valued electric potential V and magnetic potential W from certain weighted Lebesgue and Sobolev spaces, respectively. We concentrate our attention mostly to the two-dimensional case. Solvability of the direct scattering problem for $Hu = k^2u$ is investigated in a weighted Lebesgue

space. Asymptotic behavior of scattering solutions is established giving rise to scattering amplitude. First nonlinear term of the backscattering amplitude is studied in order to consider the direct and inverse Born approximation of the function $|W|^2 + V$. We discuss the recovery of shape and location of electric potential V from the knowledge of backscattering amplitude, where the direction of measurement opposes that of incidence. Numerical examples are given to illustrate the method.

Joint work with V. Serov.

Reconstruction of Riemannian manifold from boundary and interior data

TEEMU SAKSALA (*Rice University*), teemu.saksala@rice.edu

Abstract. In this talk we focus on two Geometric inverse problems, which are both related to seismic imaging using the Earthquakes. We model the Earth as a smooth and compact Riemannian manifold with or without boundary. Earthquakes are considered to be the solutions of Riemannian wave equation with interior point sources and zero initial values. We assume that there is a large amount of Earthquakes. We study two different kinds of data related to Earthquakes. We show that both of these data determine the underlying Riemannian manifold up to isometry.

We call the first data the *Distance difference data*. Here we study a closed Riemannian manifold (N, g) and we assume that there exists an open set, with smooth boundary, such that for its closure F the Riemannian structure $g|_F$ is given. We assume that for any point $x \in M = N \setminus F$ the corresponding distance difference function

$$D_x : F \times F \rightarrow \mathbb{R}, D_x(z_1, z_2) = d(x, z_1) - d(x, z_2).$$

is given.

We call the second data *Scattering Data of Internal Sources*. Here we study a smooth, compact, non trapping Riemannian manifold (\bar{M}, g) with smooth strictly convex boundary. We assume that g belongs to a certain generic class of metrics and we assume that $g|_{\partial M}$ is given. In addition we assume that for each $x \in M$ the scattering set of the point source x , that is

$$R_{\partial M}(x) := \{(\gamma_{x,\xi}(t_\xi), \dot{\gamma}_{x,\xi}(t_\xi))^T \in T\partial M : \xi \in T_x M, \|\xi\| = 1\}$$

is given. Here $\gamma_{x,\xi}$ is the unique geodesic with initial values (x, ξ) , t_ξ is the exit time of $\gamma_{x,\xi}$ and v^T , $v \in \partial TM$, stands for the tangential component to ∂M of vector v .

The talk is based on the following manuscripts:

- [1] Determination of a Riemannian manifold from the distance difference functions, in collaboration with Matti Lassas, arXiv:1510.06157
- [2] Reconstruction of a compact Riemannian manifold from the scattering data of internal sources, in collaboration with Matti Lassas and Hanming Zhou, arXiv:1708.07573

Joint work with M. Lassas and H. Zhou.