INVERSE PROBLEMS I

Place and time: In M103 on Thursday, Jan 4, at 10:30–12:00
Organizers: Mikko Salo (University of Jyväskylä)
Markus Harju (University of Oulu)
Contact email: markus.harju@oulu.fi

Geodesic X-ray transform on nontrapping manifolds

Jere Lehtonen *(University of Jyväskylä)*, jere.ta.lehtonen@jyu.fi

Abstract. We have a compact manifold with strictly convex boundary and an unknown function (suitably regular enough). It has been conjectured that the unknown function can be determined from its integrals over geodesics joining two boundary points. Together with J. Ilmavirta and M. Salo we confirmed this conjecture in the case of piecewise constant functions in dimension two. In higher dimensional case we need to assume a certain foliation condition to be satisfied too. The proof is based on rather elemantary geometric arguments. Preprint arXiv:1702.07622.

Joint work with J. Ilmavirta and M. Salo.

Boundary determination with interior data

Tommi Brander *(Technical University of Denmark)*, tobr@dtu.dk

Abstract. We consider the conductivity equation

$$-\nabla \cdot (\sigma \nabla u) = 0$$

and recover conductivity at the boundary of an object from a combination of power density interior and Cauchy boundary data, with quite arbitrary boundary data. Depending on the imaging modality, we either recover the conductivity uniquely, or we recover two alternatives, from which we must select the correct one. The argument is elementary and local. More generally, we consider the weighted p-Laplacian as a forward model and interior data $\sigma|\nabla u|^q$ and find out that single measurement specifies the boundary conductivity when $p \geq q+1$, and otherwise the measurement specifies two alternatives. There are special problems when $p = q+1$ and this is also the only case where single interior measurement in 1 dimension does not specify the conductivity on the interval.

Recvery of conductivity in the interior can be reduced to solving a weighted $p-q$ -Laplace equation. The hyperbolicity of this equation is related to the nonuniqueness of boundary determination when $p-q<1$.

Joint work with C. Li.

Direct and inverse scattering problems for the perturbed biharmonic operator

Teemu Tyni *(University of Oulu)*, teemu.tyni@oulu.fi

Abstract. In scattering problems one is interested in how some obstacle scatters in-coming waves. The aim of this talk is to gently survey our results on the

direct and inverse scattering problems for perturbed biharmonic operator. We consider the operator $H_4 := \Delta^2 + \vec{q} \cdot \nabla + V$ that contains two possibly singular and complex-valued coefficients which are assumed to be from certain (weighted) Sobolev and Lebesgue spaces. We show that there exists a unique solution to the direct scattering problem in any dimension. Then by studying the asymptotic behaviour of the solution we define our scattering data, the scattering amplitude. The inverse scattering problem is to recover the unknown coefficients of H_4 given the scattering amplitude. We show that the so-called inverse Born approximation can be used to recover the singularities in the coefficients of operator H_4. To conclude the talk we present numerical examples of the recovery procedure in 1D and 2D.

Joint work with V. Serov and M. Harju.