DISCRETE MATHEMATICS

Place and time: In M107 on Thursday, Jan 4, at 16:00–17:30
Organizers: Aleksi Saarela (University of Turku)
Ville Salo (University of Turku)
Contact emails: amsaar@utu.fi, vosalo@utu.fi

Cellular Automata and Powers of p/q

JOHAN KOPRA *(University of Turku)*, jtjkop@utu.fi

Abstract. In dynamical systems theory one often inquires about the long-term evolution of points of some system X under repeated iteration of a self-map $X : T \to T$. One example of such a system is $\mathbb{R}_+$ equipped with the map $x \to (3/2)x$, and we are particularly interested in the evolution of fractional parts of x. It is an old result of Weyl that the fractional parts of the sequence $(3/2)^n x$ $(n = 0, 1, 2, \dots)$ are uniformly distributed in $[0, 1)$ for almost all x, but little is known about what types of distribution are possible for exceptional values of x. In particular, Mahler asked in 1968 whether there exists a real number $x > 0$ such that the fractional part of $(3/2)^n x$ never becomes greater than 1/2.

We apply a cellular automaton multiplying by p/q to a generalized version of Mahler's problem. For $p > q$ we show that there exist arbitrarily small finite unions of intervals J such that the fractional parts of $(p/q)^n x$ stay indefinitely in J for some $x > 0$. In the other direction, we show that there exist finite unions of intervals K with size arbitrarily close to 1 such that the fractional part of $(p/q)^n x$ eventually drifts outside of K for every choice of $x > 0$.

Joint work with Jarkko Kari.

Identification in the Cartesian Product of Three Complete Graphs

TUOMO LEHTILÄ *(University of Turku)*, tualeh@utu.fi

Abstract. A subset C of vertices of a graph G is said to be an *identifying code* if each closed neighbourhood of a vertex has a unique intersection with C. The identifying codes were introduced first in 1998 by Karpovsky, Chakrabarty and Levitin. Since then the central question has been finding the identifying code with the smallest possible cardinality in different graph classes and these optimal codes have been found for many different graphs.

Goddard and Wash (2013) have studied identifying codes in the Cartesian product of three complete graphs of equal order, that is, in the graph $G = K_n \square K_n \square K_n$. They have conjectured that the size of an optimal identifying code in G is n^2 and provided a construction achieving this bound as well as proof for lower bound of $n^2 - n\sqrt{n}$. However, we have found a construction of cardinality $n^2 - \frac{1}{4}n$ when $n = 4^t$ and $t \in \mathbb{Z}_+$ thus, disproving their conjecture. Furthermore, we present a new lower bound of $n^2 - \frac{3}{2}n$ and give a construction of cardinality less than n^2 for each $n \geq 8$ which relies on Latin squares. Hence, we significantly improve the lower and upper bounds compared to the known ones.

Joint work with V. Junnila and T. Laihonen.

On the k-Abelian Equivalence Relation on Finitely Generated Free Monoids

MARKUS WHITELAND *(University of Turku)*, mawhit@utu.fi

Abstract. Let Σ^* be free monoid generated by the finite set Σ. An element $w = a_1 \cdots a_n \in \Sigma^*$, $a_i \in \Sigma$, is called a *word*, and $|w|$ denotes the length n of w. A word $x \in \Sigma^*$ is a *factor* of w, if $w = yxz$ for some words $y, z \in \Sigma^*$. Let $k \geq 1$. We consider an equivalence relation called *k-abelian equivalence* over Σ^* defined as follows: Two words $u, v \in \Sigma^*$ are *k-abelian equivalent*, denoted by $u \sim_k v$, if, for all non-empty words x having $|x| \leq k$, x occurs as a factor in u and v equally many times. Note that $\sim_1$ is the *abelian equivalence* of words (u and v are the same up to permutation), and that $u \sim_k v$ implies that $|u| = |v|$.

We discuss basic properties of the equivalence classes, such as the number of equivalence classes of a given length and the size of an equivalence class represented by a given word w. We also present a characterization of k-abelian equivalence in terms of rewriting. This gives a connection to regular languages, which opens new aspects of k-abelian equivalence.

Joint work with J. Cassaigne, J. Karhumäki, S. Puzynina, M. Rao.